Particle Size Distribution Representation (Part 3)

7 Rosin - Rammler - Bennett distribution (RRB):
- Also called Rosin - Rammler - Sperling - Bennett (RRSB) distribution.
- Also called Weibull distribution.
- It is probably the most well-known distribution in the cement and the mining industry.
- RRB is widely used to analyse all types of materials, crushed or not, ground or not.
- The conventional RRB function is described by:

\[
R = 100 \times e^{-\left(\frac{x}{a}\right)^m}
\]

(Main equation of RRB)

Where:
- \(R\) = mass retained in %
- \(x\) = size in microns
- \(m\) = slope of the plot
- \(a\) = size at 36,8% of particles retained (36,8 is the ratio 100/e and e is the Neper number - 2,718)
- From the equation hereabove, we obtain:

\[
\frac{100}{R} = e^{\left(\frac{x}{a}\right)^m}
\]

- Considering that we take the logs on both sides of the equation, we have:

\[
\log\left(\frac{100}{R}\right) = \left(\frac{x}{a}\right)^m \times \log e
\]

and:
- The size parameter \( a \) can be determined by classifying a given material on a mesh size \( a = x \). This substitution in the main equation hereabove will produce a constant of about 36.8\% material retained.
- The RRB representation with a log-log vs log should be a straight line.
- Example:

<table>
<thead>
<tr>
<th>Sieve in ( \mu ) (x)</th>
<th>Passing cumulated in %</th>
<th>Residue cumulated in % (R)</th>
<th>100/R</th>
<th>log(100/R)</th>
<th>log(x)</th>
<th>log.log (100/R)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.6</td>
<td>92.4</td>
<td>1,082</td>
<td>0.0343</td>
<td>0.000</td>
<td>-1.464</td>
</tr>
<tr>
<td>4</td>
<td>19.5</td>
<td>80.5</td>
<td>1,242</td>
<td>0.0942</td>
<td>0.602</td>
<td>-1.026</td>
</tr>
<tr>
<td>16</td>
<td>46.8</td>
<td>53.2</td>
<td>1,880</td>
<td>0.2741</td>
<td>1.204</td>
<td>-0.562</td>
</tr>
<tr>
<td>32</td>
<td>75</td>
<td>25</td>
<td>4,000</td>
<td>0.6021</td>
<td>1.505</td>
<td>-0.220</td>
</tr>
<tr>
<td>48</td>
<td>88.7</td>
<td>11.3</td>
<td>8,850</td>
<td>0.9469</td>
<td>1.681</td>
<td>-0.024</td>
</tr>
<tr>
<td>64</td>
<td>97.2</td>
<td>2.8</td>
<td>35,714</td>
<td>1.5528</td>
<td>1.806</td>
<td>0.191</td>
</tr>
<tr>
<td>96</td>
<td>99.7</td>
<td>0.3</td>
<td>333,333</td>
<td>2.5229</td>
<td>1.982</td>
<td>0.402</td>
</tr>
<tr>
<td>200</td>
<td>99.9</td>
<td>0.1</td>
<td>1,000,000</td>
<td>3.0000</td>
<td>2.301</td>
<td>0.477</td>
</tr>
</tbody>
</table>

- We transform the original particle size and percentages of passing data using logarithm and log-log, and we plot on the here below graphic where axis are cartesian:
- We can see that it is necessary to modify the X and Y axis in order to get a readable and intelligible representation.
- On the X-axis, for example, 0 must be replaced by 1, 1 by 10, etc...
- On the Y-axis, -1,5 must be replaced by 7% of residue cumulated, -1 by 20,6%, etc...
- When the RRB distribution is plotted (blue line in the graph hereabove), it is still necessary to calculate a trendline (or a linear regression) in order to know the slope of the PSD (particle size distribution).
- In the example, m = 0,9055 with a normal correlation.
- The slope of the PSD of a cement, for example, is an important factor.
- More the slope is higher and tighter is the PSD.
- A tighter PSD means less superfines particles (< 3 microns) and less coarser particles (> 32 microns).
- A tighter PSD can be obtained with a good separator.

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- The RRB calculator of the website doesn’t have the right scale due to the limitations of the Excel converter.
- At the contrary, the calculator which is available in the Tromp RRB Kit software represents the real log-log vs log scale (like the special RRB paper).
- Graphic with the right scales and the original values:

8 **Log - Normal distribution:**

- Also known as Galton distribution.
- In the probability area, a lognormal distribution, or lognormal, is the probability distribution of a random variable x whose log x logarithm follows a normal distribution.
- The equation of the log-normal distribution is the following:
Where:
- \( z \) = approximate polynomial function of the inverse function of integral probability
- \( D \) = particle size dimension in µm
- \( D_{50} \) = mean geometric diameter in µm
- \( \sigma \) = standard deviation
- Parameters are \( D_{50} \) and \( \sigma \).
- The Y-axis is a probability of passing cumulated in %, then it uses a probability scale.
- As Gaudin - Schuhmann and Rosin - Rammler - Bennett models may be linearized modifying the original values in log or log.log, one can compare them.
- At the contrary, it is not possible with the Log - Normal model.
- Following this, Lawless (1978) developed equations to obtain a linear correlation coefficient for lognormal distribution (\( * \)).
- These equations have an insignificant absolute error and are the following:

\[
\text{For } 0 < x \leq 50\% \\
\begin{align*}
\kappa &= \sqrt{\ln \left( \frac{1}{x^2} \right)} \\
z &= -\kappa + a + b \cdot \kappa + c \cdot \kappa^2 \\
& \quad \left/ \left( 1 + d \cdot \kappa + e \cdot \kappa^2 + f \cdot \kappa^3 \right) \right.
\end{align*}
\]

\[
\text{For } 50\% < x < 100\% \\
\begin{align*}
\kappa &= \sqrt{\ln \left( \frac{1}{(1-x)^2} \right)} \\
z &= \kappa - a + b \cdot \kappa + c \cdot \kappa^2 \\
& \quad \left/ \left( 1 + d \cdot \kappa + e \cdot \kappa^2 + f \cdot \kappa^3 \right) \right.
\end{align*}
\]

Where:
- \( x \) = percentage of passing cumulated for a given size (\( * \))

\( * \) Be careful because the equations don't accept a value of 0% or 100%

\( t \) = a parameter to solve \( z \)

\( z \) = approximate polynomial function which will replace \( x \)

a, b, c, d, e and f = constants with the following values:

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- Example:

<table>
<thead>
<tr>
<th>Sieve in μm (D)</th>
<th>Passing cumulated in % (x)</th>
<th>ln(D)</th>
<th>t</th>
<th>z</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7.6</td>
<td>0.00</td>
<td>2.27</td>
<td>-1.43275796</td>
</tr>
<tr>
<td>4</td>
<td>19.5</td>
<td>1.39</td>
<td>1.808</td>
<td>-0.85945552</td>
</tr>
<tr>
<td>16</td>
<td>46.8</td>
<td>2.77</td>
<td>1.232</td>
<td>-0.08008182</td>
</tr>
<tr>
<td>32</td>
<td>75</td>
<td>3.47</td>
<td>1.665</td>
<td>0.67417561</td>
</tr>
<tr>
<td>48</td>
<td>88.7</td>
<td>3.87</td>
<td>2.088</td>
<td>1.21064581</td>
</tr>
<tr>
<td>64</td>
<td>97.2</td>
<td>4.16</td>
<td>2.674</td>
<td>1.91145128</td>
</tr>
<tr>
<td>96</td>
<td>99.7</td>
<td>4.56</td>
<td>3.409</td>
<td>2.74814884</td>
</tr>
<tr>
<td>200</td>
<td>99.9</td>
<td>5.30</td>
<td>3.717</td>
<td>3.09051634</td>
</tr>
</tbody>
</table>

- We transform the original particle size and percentages of passing data using natural logarithm and z, and we plot on the here below graphic where axis are cartesian:

- Graphic with the right scales and the original value:
Example of RRSB graph and results obtained in the GK software available at [www.thecementgrindingoffice.com/cvs.html](http://www.thecementgrindingoffice.com/cvs.html)