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Particle Size Distribution Representation (Part 3)

7 Rosin - Rammler - Bennett distribution (RRB):

- Also called Rosin - Rammler - Sperling - Bennett (RRSB) distribution.
- Also called Weibull distribution.
- It is probably the most well-known distribution in the cement and the mining industry.
- RRB is widely used to analyse all types of materials, crushed or not, ground or not.
- The conventional RRB function is described by:

$$R = 100 \times e^{-\left(\frac{x}{a}\right)^m}$$ (Main equation of RRB)

Where:

R = mass retained in %

x = size in microns

m = slope of the plot

a = size at 36,8% of particles retained (36,8 is the ratio 100/e and e is the Neper number - 2,718)

- From the equation hereabove, we obtain:

$$\frac{100}{R} = e^{\left(\frac{x}{a}\right)^m}$$

- Considering that we take the logs on both sides of the equation, we have:

$$\log\left(\frac{100}{R}\right) = \left(\frac{x}{a}\right)^m \times \log e$$

and:

$$\log\log\left(\frac{100}{R}\right) = m \times \log\left(\frac{x}{a}\right) + \log\log e$$

and:

$$\log\log\left(\frac{100}{R}\right) = m \times \log x - m \times \log a + \log\log e$$

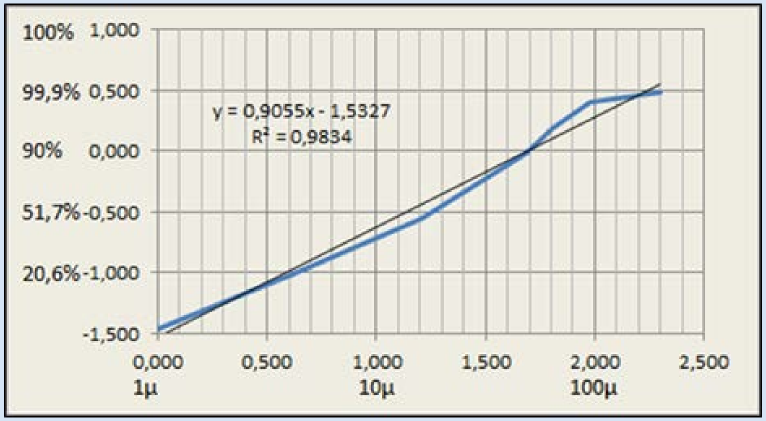
and:

$$\log\log\left(\frac{100}{R}\right) = m \times \log x + C$$

- The size parameter a can be determined by classifying a given material on a mesh size a = x.
  This substitution in the main equation hereabove will produce a constant of about 36.8% material retained.
- The RRB representation with a log.log vs log should be a straight line.
- Example:

Sieve in µ (x)	Passing cumulated in %	Residue cumulated in % (R)	100/R	log(100/R)	log(x)	log.log (100/R)
1	7,6	92,4	1,082	0,0343	0,000	-1,464
4	19,5	80,5	1,242	0,0942	0,602	-1,026
16	46,8	53,2	1,880	0,2741	1,204	-0,562
32	75	25	4,000	0,6021	1,505	-0,220
48	88,7	11,3	8,850	0,9469	1,681	-0,024
64	97,2	2,8	35,714	1,5528	1,806	0,191
96	99,7	0,3	333,333	2,5229	1,982	0,402
200	99,9	0,1	1000,000	3,0000	2,301	0,477

- We transform the original particle size and percentages of passing data using logarithm and log.log, and we plot on the here below graphic where axis are cartesian:



- We can see that it is necessary to modify the X and Y axis in order to get a readable and intelligible representation.
- On the X-axis, for example, 0 must be replaced by 1, 1 by 10, etc...
- On the Y-axis, -1,5 must be replaced by 7% of residue cumulated, -1 by 20,6%, etc...
- When the RRB distribution is plotted (blue line in the graph hereabove), it is still necessary to calculate a trendline (or a linear regression) in order to know the slope of the PSD (particle size distribution).
- In the example, m = 0,9055 with a normal correlation.
- The slope of the PSD of a cement, for example, is an important factor.
- More the slope is higher and tighter is the PSD.
- A tighter PSD means less superfines particles (< 3 microns) and less coarser particles (> 32 microns).
- A tighter PSD can be obtained with a good separator.

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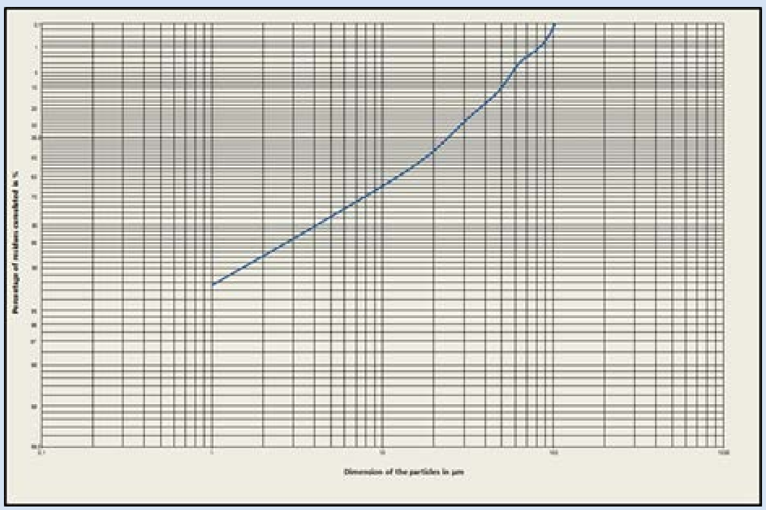
- **The RRB calculator of the website doesn't have the right scale due to the limitations of the Excel converter.**
  http://www.thecementgrindingoffice.com/rrb.html
- **At the contrary, the calculator which is available in the Tromp RRB Kit software represents the real log.log vs log scale (like the special RRB paper).**
  http://www.thecementgrindingoffice.com/cvs.html
- Graphic with the right scales and the original values:



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8 Log - Normal distribution:

- Also known as Galton distribution.
- In the probability area, a lognormal distribution, or lognormal, is the probability distribution of a random variable x whose logx logarithm follows a normal distribution.
- The equation of the log-normal distribution is the following:

$$z = \frac{\ln\left(\frac{D}{D_{50}}\right)}{\sqrt{2}.\ln\sigma}$$

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-z^2}.dz$$

Where:
z = approximate polynomial function of the inverse function of integral probability
D = particle size dimension in µm
D_{50} = mean geometric diameter in µm
σ = standard deviation

- Parameters are D_{50} and σ.
- The Y-axis is a probability of passing cumulated in %, then it uses a probability scale.
- As Gaudin - Schuhmann and Rosin - Rammler - Bennett models may be linearized modifying the original values in log or log.log, one can compare them.
- At the contrary, it is not possible with the Log - Normal model.
- Following this, Lawless (1978) developed equations to obtain a linear correlation coefficient for lognormal distribution (*).
- These equations have an insignificant absolute error and are the following:

For $0 < x \leq 50\%$

$$t = \sqrt{\ln\left(\frac{1}{x^2}\right)}$$

$$z = -t + \frac{a + b.t + c.t^2}{1 + d.t + e.t^2 + f.t^3}$$

For $50\% < x < 100\%$

$$t = \sqrt{\ln\left(\frac{1}{(1-x)^2}\right)}$$

$$z = t - \frac{a + b.t + c.t^2}{1 + d.t + e.t^2 + f.t^3}$$

Where:
x = percentage of passing cumulated for a given size (*)
(*) Be carreful because the equations don't accept a value of 0% or 100%
t = a parameter to solve z
z = approximate polynomial function which will replace x
a, b, c, d, e and f = constants with the following values:

a	2,51557
b	0,802853
c	0,010328
d	1,432788
e	0,189269
f	0,001308

- Reference: Log-normal model linearization for particle size distribution, Laércio Montovani Frare, Marcelino Luiz Gimenes, Nehemias Curvelo Pereira e Elisabete Scolin Mendes (Departamento de Engenharia Química, Universidade Estadual de Maringá, Av. Colombo, 5790, 87020-900, Maringá-Paraná, Brasil).

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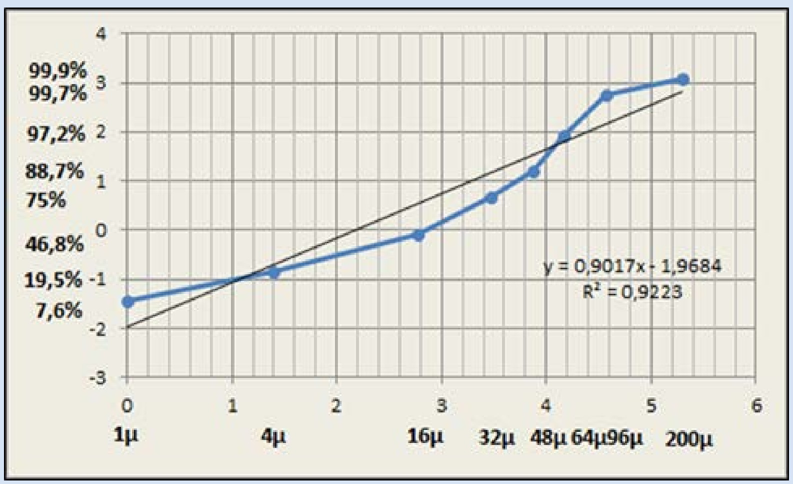
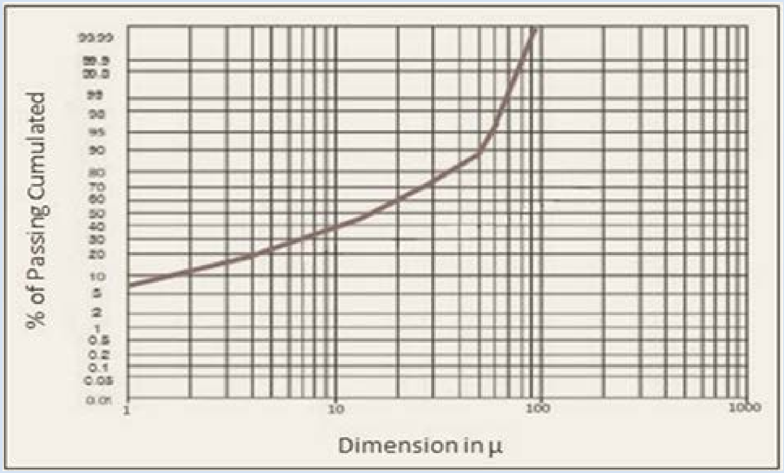
- Example:

Sieve in µ (D)	Passing cumulated in % (x)	ln(D)	t	z
1	7,6	0,00	2,270	-1,43275796
4	19,5	1,39	1,808	-0,85945552
16	46,8	2,77	1,232	-0,08008182
32	75	3,47	1,665	0,67417561
48	88,7	3,87	2,088	1,21084581
64	97,2	4,16	2,674	1,91145128
96	99,7	4,56	3,409	2,74814884
200	99,9	5,30	3,717	3,09051634

- We transform the original particle size and percentages of passing data using natural logarithm and z, and we plot on the here below graphic where axis are cartesian:



- Graphic with the right scales and the original value:



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Example of RRSB graph and results obtained in the GK software available at www.thecementgrindingoffice.com/cvs.html

RRSB with right scales